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Black holes, brick walls and the Boulware state

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Abstract

The brick-wall model seeks to explain the Bekenstein-Hawking entropy as a wall-contribution to the thermal energy of ambient quantum fields raised to the Hawking temperature. Reservations have been expressed concerning the self-consistency of this model. For example, it predicts large thermal energy densities near the wall, producing a substantial mass-correction and, presumably, a large gravitational back-reaction. We re-examine this model and conclude that these reservations are unfounded once the ground state—the Boulware state—is correctly identified. We argue that the brick-wall model and the Gibbons-Hawking instanton (which ascribes a topological origin to the Bekenstein-Hawking entropy) are mutually exclusive, alternative descriptions (complementary in the sense of Bohr) of the same physics.

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I. INTRODUCTION

The prescription

$$S_{BH} = \frac{1}{4} A/l_{pl}^2 \tag{1.1}$$

for assigning a "Bekenstein-Hawking entropy" S_{BH} to a black hole of surface area A was first inferred in the mid-1970s from the formal similarities between black hole dynamics and thermodynamics [1], combined with Hawking's discovery [2] that black hole radiate thermally with a characteristic (Hawking) temperature

$$T_H = \hbar\kappa_0/2\pi, \tag{1.2}$$

where κ_0 is the surface gravity. That S_{BH} is a genuine thermodynamical entropy is given credence by the "generalized second law" [3,4], which states that the sum of S_{BH} and the entropy of surrounding matter is non-decreasing in any (quasi-static) interaction.

Much work and discussion have been devoted to the problem of deriving and understanding the enigmatic formula (1.1) in a more direct and fundamental way.

A statistical derivation of (1.1) for stationary black holes, using analytic continuation to the Euclidean sector and imposing a Matsubara period T_H^{-1} on Euclidean time, was developed by Gibbons and Hawking [5] in 1977. According to their derivation, S_{BH} emerges already at zero-loop order, as a contribution to the partition function from the extrinsic-curvature boundary terms which accompany the Einstein-Hilbert gravitational action. (Only the outer boundary at infinity actually contributes. There is no inner boundary, because the horizon is represented by a regular point in the Euclidean sector of a non-extremal black hole with the above periodic identification, corresponding to the Hartle-Hawking state for ambient quantum fields.)

Unfortunately, the Euclidean approach provides no insight into the dynamical origin of S_{BH}. On the contrary, it appears to suggest that one has to think of S_{BH} as having a topological origin in some sense.

Interesting and suggestive, but still short of ideal from a physical point of view, are the interpretations which require an appeal to the past or future history of the black hole, or to ensembles of such histories; for instance, "$\exp(S_{BH})$ represents the number of quantum-mechanically distinct ways in which the black hole could have been made" [3], or those which relate S_{BH} to the entropy of the evaporation products [6,7].

In thermodynamics, entropy is a state function with a definite value at each moment of time (at least for quasi-stationary, near-equilibrium processes). Correspondingly, one would ideally like to have a direct physical understanding of S_{BH} for a specific black hole at a given moment in terms of the dynamical degrees of freedom existing at that moment, together with an explanation of how it comes to have the simple universal form (1.1), independent of the nature and number of the fundamental fields and all details of the microphysics [8].

An interpretation which holds promise of meeting these requirements is that S_{BH} is entanglement entropy, associated with modes and correlations hidden from external observers by the presence of the horizon. If the black hole originates from a pure state, there is perfect correlation between the internal and external modes, and the entanglement entropy can therefore equally well be found by counting external modes. A program of this type was first clearly formulated by Bombelli, Koul, Lee and Sorkin [9] in a 1986 paper which attracted little notice at the time. It was independently re-initiated by Srednicki [10] and by Frolov and Novikov [8] in 1993.

It turns out, remarkably, that the entanglement entropy is proportional to the area of the dividing wall. This is a robust result which holds not only for black hole horizons [8,11], but also for cavities artificially cut out of Minkowski space [9,10,12]. However, the coefficient of proportionality is formally infinite, corresponding to the fact that modes of arbitrarily small wavelength can exist close to the horizon or partition. (This, incidentally, yields the correct answer—infinity—for the entropy of a classical black hole.) But an infinitely sharp boundary violates quantum mechanics. Quantum fluctuations will prevent events closer to the horizon than about a Planck length l_{pl} from being seen on the outside. Introducing an effective cutoff of this order reproduces the Bekenstein-Hawking formula (1.1) with a

coefficient of the right order of magnitude.

The entanglement interpretation seems to be implicit in, and is certainly closely related to a pioneering calculation done by Gerard 'tHooft [13] in 1985. He considered the statistical thermodynamics of quantum fields in the Hartle-Hawking state (i.e. having the Hawking temperature T_H at large radii) propagating on a fixed Schwarzschild background of mass M. To control divergences, 'tHooft introduced a "brick wall"—actually a static spherical mirror at which the fields are required to satisfy Dirichlet or Neumann boundary conditions—with radius a little larger than the gravitational radius $2M$. He found, in addition to the expected volume-dependent thermodynamical quantities describing hot fields in a nearly flat space, additional wall contributions proportional to the area. These contributions are, however, also proportional to α^{-2}, where α is the proper altitude of the wall above the gravitational radius, and thus diverge in the limit $\alpha \to 0$. For a specific choice of α (which depends on the number of fields, etc., but is generally of order l_{pl}), 'tHooft was able to recover the Bekenstein-Hawking formula (1.1) with the correct coefficient.

However, this calculation raises a number of questions which have caused many, including 'tHooft himself, to have reservations about its validity and consistency.

(a) S_{BH} is here obtained as a one-loop effect, originating from thermal excitations of the quantum fields. Does this material contribution to S_{BH} have to be *added* to the zero-loop Gibbons-Hawking contribution which arises from the gravitational part of the action and already by itself accounts for the full value of S_{BH}? [14]

(b) The ambient quantum fields were assumed to be in the Hartle-Hawking state. Their stress-energy should therefore be bounded (of order M^{-4} in Planck units) near the gravitational radius, and negligibly small for large masses. However, 'tHooft's calculation assigns to them enormous (Planck-level) energy densities near the wall.

(c) The integrated field energy gives a wall contribution to the mass

$$\Delta M = \frac{3}{8} M \tag{1.3}$$

when α is adjusted to give the correct value of S_{BH}. This suggests a substantial gravitational back-reaction [13] and that the assumption of a fixed geometrical background may be inconsistent [14–16].

Our main purpose in this paper is to point out that these difficulties are only apparent and easy to resolve. The basic remark is that the *brick-wall model strictly interpreted does not represent a black hole.* It represents the exterior of a starlike object with a reflecting surface, compressed to nearly (but not quite) its gravitational radius. The ground state for quantum fields propagating around this star is not the Hartle-Hawking state but the Boulware state [17], corresponding to zero temperature, which has a quite different behavior near the gravitational radius.

The Boulware state in a static spherical geometry is defined as being free of modes having positive frequency with respect to the conventional Schwarzschild time t. At infinity it approaches the Minkowski vacuum with zero stress-energy. At smaller radii, vacuum polarization induces a non-vanishing stress-energy, which diverges near a brick wall skirting the gravitational radius. The asymptotic behavior is that of a thermal stress-energy—the characteristic temperature is the local acceleration temperature for static observers, diverging near the gravitational radius—*but with the opposite sign.* The Boulware state is energetically depressed below the vacuum. (We shall reserve the word "vacuum" for a condition of zero stress-energy; in general, it is not a quantum state.)

The quantum fields with temperature T_H at infinity which 'tHooft introduced into his brick wall model have a local temperature and a thermal stress-energy which also diverge near the gravitational radius. To obtain the total (gravitating) stress-energy, one must add the contributions of the (Boulware) ground state and the thermal excitations. The diverging parts of these contributions cancel. The sum is bounded, of order M^{-4} near the gravitational radius (hence small for large masses), and in the limit $\alpha \to 0$ indistinguishable from the Hartle-Hawking stress-energy.

Thus, it is perfectly legitimate to neglect back-reaction in the brick wall model. With

the proper identification of the ground state, problems (b) and (c) above resolve themselves.

The same basic observation resolves problem (a). In the Euclidean sector of the brick-wall spacetime (with Matsubara period T_H^{-1}), it is not true that there is no inner boundary. The inner boundary is the brick wall itself, and its boundary contribution to the Euclidean action cancels that of the outer boundary at infinity. So the Gibbons-Hawking zero-loop contribution is now zero. The "geometrical entropy" of the Gibbons-Hawking "instanton" thus provides an alternative, complementary description of S_{BH}, not a supplementary contribution to the entropy of thermal excitations as calculated from the brick wall model.

In Sec. II we summarize essential properties of the Boulware and Hartle-Hawking states that play a role in this paper. In Sec. III we sketch the physical essence of the brick-wall model by using a particle description of quantum fields. A systematic treatment of the model is deferred to Sec. IV, in which the results in Sec. III are rigorously derived from the quantum field theory in curved spacetimes. Sec. V is devoted to summarize this paper. In Appendix A, for completeness, we apply the so-called on-shell method to the brick wall model and show that in the on-shell method we might miss some physical degrees of freedom. Hence, we do not adopt the on-shell method in the main body of this paper.

II. THE BOULWARE AND HARTLE-HAWKING STATES

It is useful to begin by summarizing briefly the essential properties of the quantum states that will play a role in our discussion.

In a curved spacetime there is no unique choice of time coordinate. Different choices lead to different definitions of positive-frequency modes and different ground states.

In any static spacetime with static (Killing) time parameter t, the Boulware state $|B\rangle$ is the one annuled by the annihilation operators a_{Kill} associated with "Killing modes" (positive-frequency in t). In an asymptotically flat space, $|B\rangle$ approaches the Minkowski vacuum at infinity.

In the spacetime of a stationary eternal black hole, the Hartle-Hawking state $|HH\rangle$

is the one annuled by a_{Krus}, the annihilation operators associated with "Kruskal modes" (positive-frequency in the Kruskal lightlike coordinates U, V). This state appears empty of "particles" to free falling observers at the horizon, and its stress-energy is bounded there (not quite zero, because of irremovable vacuum polarization effects).

If, just for illustrative purpose, we consider a $(1+1)$-dimensional spacetime, it is easy to give concrete form to these remarks. We consider a spacetime with metric

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)}, \tag{2.1}$$

and denote by $\kappa(r)$ the redshifted gravitational force, i.e., the upward acceleration $a(r)$ of a stationary test-particle reduced by the redshift factor $f^{1/2}(r)$, so that $\kappa(r) = \frac{1}{2}f'(r)$. A horizon is characterized by $r = r_0$, $f(r_0) = 0$, and its surface gravity defined by $\kappa_0 = \frac{1}{2}f'(r_0)$.

Quantum effects induce an effective quantum stress-energy T_{ab} $(a, b, \cdots = r, t)$ in the background geometry (2.1). If we assume no net energy flux ($T_t^r = 0$)—thus excluding the Unruh state—T_{ab} is completely specified by a quantum energy density $\rho = -T_t^t$ and pressure $P = T_r^r$. These are completely determined (up to a boundary condition) by the conservation law $T_{a;b}^b = 0$ and the trace anomaly, which is

$$T_a^a = \frac{\hbar}{24\pi} R \tag{2.2}$$

for a massless scalar field, with $R = -f''(r)$ for the metric (2.1). Integration gives

$$f(r)P(r) = -\frac{\hbar}{24\pi}(\kappa^2(r) + const.). \tag{2.3}$$

Different choices of the constant of integration correspond to different boundary conditions, i.e., to different quantum states.

For the Hartle-Hawking state, we require P and ρ to be bounded at the horizon $r = r_0$, giving

$$\begin{aligned} P_{HH} &= \frac{\hbar}{24\pi}\frac{\kappa_0^2 - \kappa^2(r)}{f(r)}, \\ \rho_{HH} &= P_{HH} + \frac{\hbar}{24\pi}f''(r). \end{aligned} \tag{2.4}$$

When $r \to \infty$ this reduces to (setting $f(r) \to 1$)

$$\rho_{HH} \simeq P_{HH} = \frac{\pi}{6\hbar} T_H^2,$$
$$T_H = \hbar \kappa_0 / 2\pi, \tag{2.5}$$

which is appropriate for one-dimensional scalar radiation at the Hawking temperature T_H.

For the Boulware state, the boundary condition is $P = \rho = 0$ when $r = \infty$. The integration constant in (2.3) must vanish, and we find

$$P_B = -\frac{\hbar}{24\pi} \frac{\kappa^2(r)}{f(r)},$$
$$\rho_B = P_B + \frac{\hbar}{24\pi} f''(r). \tag{2.6}$$

If a horizon were present, ρ_B and P_B would diverge there to $-\infty$.

For the difference of these two stress tensors,

$$\Delta T_a^b = (T_a^b)_{HH} - (T_a^b)_B, \tag{2.7}$$

(2.4) and (2.6) give the exactly thermal form

$$\Delta P = \Delta \rho = \frac{\pi}{6\hbar} T^2(r), \tag{2.8}$$

where $T(r) = T_H / \sqrt{f(r)}$ is the local temperature in the Hartle-Hawking state. We recall that thermal equilibrium in any static gravitational field requires the local temperature T to rise with depth in accordance with Tolman's law [18]

$$T\sqrt{-g_{00}} = const. \tag{2.9}$$

We have found, for this $(1+1)$-dimensional example, that the Hartle-Hawking state is thermally excited above the zero-temperature (Boulware) ground state to a local temperature $T(r)$ which grows without bound near the horizon. Nevertheless, it is the Hartle-Hawking state which best approximates what a gravitational theorist would call a "vacuum" at the horizon.

These remarks remain at least qualitatively valid in $(3+1)$-dimensions, with obvious changes arising from the dimensionality. In particular, the $(3+1)$-dimensional analogue of (2.8) for a massless scalar field,

$$3\Delta P \simeq \Delta\rho \simeq \frac{\pi^2}{30\hbar^3}T^4(r), \tag{2.10}$$

holds to a very good approximation, both far from the black hole and near the horizon. In the intermediate zone there are deviations, but they always remain bounded [19], and will not affect our considerations.

III. BRICK-WALL MODEL

We shall briefly sketch the physical essence of the brick-wall model. (A systematic treatment is deferred to Sec. IV)

We wish to study the thermodynamics of hot quantum fields confined to the outside of a spherical star with a perfectly reflecting surface whose radius r_1 is a little larger than its gravitational radius r_0. To keep the total field energy bounded, we suppose the system enclosed in a spherical container of radius $L \gg r_1$.

It will be sufficiently general to assume for the geometry outside the star a spherical background metric of the form

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega^2. \tag{3.1}$$

This includes as special cases the Schwarzschild, Reissner-Nordström and de Sitter geometries, or any combination of these.

Into this space we introduce a collection of quantum fields, raised to some temperature T_∞ at large distances, and in thermal equilibrium. The local temperature $T(r)$ is then given by Tolman's law (2.9),

$$T(r) = T_\infty f^{-1/2} \tag{3.2}$$

and becomes very large when $r \to r_1 = r_0 + \Delta r$. We shall presently identify T_∞ with the Hawking temperature T_H of the horizon $r = r_0$ of the exterior metric (3.1), continued (illegitimately) into the internal domain $r < r_1$.

Characteristic wavelengths λ of this radiation are small compared to other relevant length-scales (curvature, size of container) in the regions of interest to us. Near the star's surface,

$$\lambda \sim \hbar/T = f^{1/2}\hbar/T_\infty \ll r_0. \tag{3.3}$$

Elsewhere in the large container, at large distances from the star,

$$f \simeq 1, \quad \lambda \simeq \hbar/T_\infty \sim r_0 \ll L. \tag{3.4}$$

Therefore, a particle description should be a good approximation to the statistical thermodynamics of the fields (Equivalently, one can arrive at this conclusion by considering the WKB solution to the wave equation, cf. 'tHooft [13] and Sec. IV)

For particles of rest-mass m, energy E, 3-momentum p and 3-velocity v as viewed by a local stationary observer, the energy density ρ, pressure P and entropy density s are given by the standard expressions

$$\begin{aligned} \rho &= \mathcal{N}\int_0^\infty \frac{E}{e^{\beta E}-\epsilon}\frac{4\pi p^2 dp}{h^3}, \\ P &= \frac{\mathcal{N}}{3}\int_0^\infty \frac{vp}{e^{\beta E}-\epsilon}\frac{4\pi p^2 dp}{h^3}, \\ s &= \beta(\rho + P). \end{aligned} \tag{3.5}$$

Here, as usual,

$$E^2 - p^2 = m^2, \quad v = p/E, \quad \beta = T^{-1}; \tag{3.6}$$

ϵ is $+1$ for bosons and -1 for fermions and the factor $\mathcal{N}$ takes care of helicities and the number of particle species. The total entropy is given by the integral

$$S = \int_{r_1}^{L} s(r) 4\pi r^2 dr/\sqrt{f}, \tag{3.7}$$

where we have taken account of the proper volume element as given by the metric (3.1). The factor $f^{-1/2}$ does not, however, appear in the integral for the gravitational mass of the thermal excitations [20] (It is canceled, roughly speaking, by negative gravitational potential energy):

$$\Delta M_{therm} = \int_{r_1}^{L} \rho(r) 4\pi r^2 dr. \tag{3.8}$$

The integrals (3.7) and (3.8) are dominated by two contributions for large container radius L and for small $\Delta r = r_1 - r_0$:

(a) A volume term, proportional to $\frac{4}{3}\pi L^3$, representing the entropy and mass-energy of a homogeneous quantum gas in a flat space (since $f \simeq 1$ almost everywhere in the container if $L/r_0 \to \infty$) at a uniform temperature T_∞. This is the result that would have been expected, and we do not need to consider it in detail.

(b) Of more interest is the contribution of gas near the inner wall $r = r_1$, which we now proceed to study further. We shall find that it is proportional to the wall area, and diverging like $(\Delta r)^{-1}$ when $\Delta r \to 0$.

Because of the high local temperatures T near the wall for small Δr, we may insert the ultrarelativistic approximations

$$E \gg m, \quad p \simeq E, \quad v \simeq 1$$

into the integrals (3.5). This gives

$$P \simeq \frac{1}{3}\rho \simeq \frac{\mathcal{N}}{6\pi^2} T^4 \int_0^\infty \frac{x^3 dx}{e^x - \epsilon} \tag{3.9}$$

in Planck units ($h = 2\pi\hbar = 2\pi$). The purely numerical integral has the value 3! multiplied by 1, $\pi^4/90$ and $\frac{7}{8}\pi^4/90$ for $\epsilon = 0, 1$ and 1 respectively, and we shall adopt 3!, absorbing any small discrepancy into $\mathcal{N}$. Then, from (3.5),

$$\rho = \frac{3\mathcal{N}}{\pi^2} T^4, \quad s = \frac{4\mathcal{N}}{\pi^2} T^3 \tag{3.10}$$

in terms of the local temperature T given by (3.2).

Substituting (3.10) into (3.7) gives for the wall contribution to the total entropy

$$S_{wall} = \frac{4\mathcal{N}}{\pi^2} 4\pi r_1^2 T_\infty^3 \int_{r_1}^{r_1+\delta} \frac{dr}{f^2(r)}, \tag{3.11}$$

where δ is an arbitrary small length subject to $\Delta r \ll \delta \ll r_1$. It is useful to express this result in terms of the proper altitude α of the inner wall above the horizon $r = r_0$ of the exterior geometry (3.1). (Since (3.1) only applies for $r > r_1$, the physical space does not, of course, contain any horizon.) We assume that $f(r)$ has a (simple) zero for $r = r_0$, so we can write

$$f(r) \simeq 2\kappa_0 (r - r_0), \quad \kappa_0 = \frac{1}{2} f'(r_0) \neq 0 \quad (r \to r_0), \tag{3.12}$$

where κ_0 is the surface gravity. Then

$$\alpha = \int_{r_0}^{r_1} f^{-1/2} dr \quad \Rightarrow \quad \Delta r = \frac{1}{2} \kappa_0 \alpha^2, \tag{3.13}$$

and (3.11) can be written

$$S_{wall} = \frac{\mathcal{N}}{90\pi\alpha^2} \left(\frac{T_\infty}{\kappa_0/2\pi} \right)^3 \frac{1}{4} A \tag{3.14}$$

in Planck units, where $A = 4\pi r_1^2$ is the wall area.

Similarly, we find from (3.8) and (3.10) that thermal excitations near the wall contribute

$$\Delta M_{them,wall} = \frac{\mathcal{N}}{480\pi\alpha^2} \left(\frac{T_\infty}{\kappa_0/2\pi} \right)^3 A T_\infty \tag{3.15}$$

to the gravitational mass of the system.

The wall contribution to the free energy

$$F = \Delta M - T_\infty S \tag{3.16}$$

is

$$F_{wall} = -\frac{\mathcal{N}}{1440\pi\alpha^2} \left(\frac{T_\infty}{\kappa_0/2\pi} \right)^3 A T_\infty . \tag{3.17}$$

The entropy is recoverable from the free energy by the standard prescription

$$S_{wall} = -\partial F_{wall}/\partial T_\infty. \tag{3.18}$$

(Observe that this is an "off-shell" prescription [21]: the geometrical quantities A, α and, in particular, the surface gravity κ_0 are kept fixed when the temperature is varied in (3.17).)

Following 'tHooft [13], we now introduce a crude cutoff to allow for quantum-gravity fluctuations by fixing the wall altitude α so that

$$S_{wall} = S_{BH}, \quad \text{when} \quad T_\infty = T_H, \tag{3.19}$$

where the Bekenstein-Hawking entropy S_{BH} and Hawking temperature T_H are defined to be the *purely geometrical* quantities defined by (1.1) and (1.2) in terms of the wall's area A and redshifted acceleration (= surface gravity) κ_0. From (3.19) and (3.14), restoring conventional units for a moment, we find

$$\alpha = l_{pl}\sqrt{\mathcal{N}/90\pi}, \tag{3.20}$$

so that α is very near the Planck length if the effective number $\mathcal{N}$ of basic quantum fields in nature is on the order of 300.

It is significant and crucial that the normalization (3.20) is *universal*, depending only on fundamental physics, and independent of the mechanical and geometrical characteristics of the system.

With α fixed by (3.20), the wall's free energy (3.16) becomes

$$F_{wall} = -\frac{1}{16}\left(\frac{T_\infty}{T_H}\right)^3 A T_\infty. \tag{3.21}$$

This "off-shell" formula expresses F_{wall} in terms of three independent variables: the temperature T_∞ and the geometrical characteristics A and T_H. From (3.21) we can obtain the wall entropy either from the thermodynamical Gibbs relation (3.18) (with T_∞ set equal to T_H *after* differentiation), or from the Gibbs-Duhem formula (3.16) which is equivalent to the statistical-mechanical definition $S = -\mathbf{Tr}(\rho \ln \rho)$. Thus the distinction [21] between

"thermodynamical" and "statistical" entropies disappears in this formulation, because the geometrical and thermal variables are kept independent.

The wall's thermal mass-energy is given "on-shell" ($T_\infty = T_H$) by

$$\Delta M_{therm,wall} = \frac{3}{16} A T_H \tag{3.22}$$

according to (3.15) and (3.20). For a wall skirting a Schwarzschild horizon, so that $T_H = (8\pi M)^{-1}$, this reduces to 'tHooft's result (1.3).

As already noted, thermal energy is not the only source of the wall's mass. Quantum fields outside the wall have as their ground state the Boulware state, which has a negative energy density growing to Planck levels near the wall. On shell, this very nearly cancels the thermal energy density (3.10); their sum is, in fact, the Hartle-Hawking value (cf. (2.7) and (2.10)):

$$(T_\mu^\nu)_{therm,T_\infty=T_H} + (T_\mu^\nu)_B = (T_\mu^\nu)_{HH}, \tag{3.23}$$

which remains bounded near horizons, and integrates virtually to zero for a very thin layer near the wall. The total gravitational mass of the wall is thus, from (3.15) and (3.20),

$$\begin{aligned}(\Delta M)_{wall} &= (\Delta M)_{therm,wall} + (\Delta M)_{B,wall} \\ &= \frac{3}{16} A T_H \left((T_\infty / T_H)^4 - 1 \right), \end{aligned} \tag{3.24}$$

which vanishes on shell. For a central mass which is large in Planck units, there is no appreciable back-reaction of material near the wall on the background geometry (3.1).

We may conclude that many earlier concerns [13–15] were unnecessary: 'tHooft's brick wall model does provide a perfectly self-consistent description of a configuration which is indistinguishable from a black hole to outside observers, and which accounts for the Bekenstein-Hawking entropy purely as thermal entropy of quantum fields at the Hawking temperature (i.e. in the Hartle-Hawking state), providing one accepts the ad hoc but plausible ansatz (3.20) for a Planck-length cutoff near the horizon.

The model does, however, present us with a feature which is theoretically possible but appears strange and counterintuitive from a gravitational theorist's point of view. Although

the wall is insubstantial (just like a horizon)—i.e., space there is practically a vacuum and the local curvature low—it is nevertheless the repository of all of the Bekenstein-Hawking entropy in the model.

It has been argued [8] that this is just what might be expected of black hole entropy in the entanglement picture. Entanglement will arise from virtual pair-creation in which one partner is "invisible" and the other "visible" (although only temporarily—nearly all get reflected back off the potential barrier). Such virtual pairs are all created very near the horizon. Thus, on this picture, the entanglement entropy (and its divergence) arises almost entirely from the strong correlation between nearby field variables on the two sides of the partition, an effect already present in flat space [22].

An alternative (but not necessarily incompatible) possibility is that the concentration of entropy at the wall is an artifact of the model or of the choice of Fock representation (based on a static observer's definition of positive frequency). The boundary condition of perfect reflectivity at the wall has no black hole counterpart. Moreover, one may well suspect that localization of entanglement entropy is not an entirely well-defined concept [23] or invariant under changes of the Fock representation.

IV. THE BRICK WALL MODEL REVISITED

In the previous section, we have investigated the statistical mechanics of quantum fields in the region $r_1 < r < L$ of the spherical background (3.1) with the Dirichlet boundary condition at the boundaries. By using the particle description with the local temperature given by the Tolman's law, we have obtained the inner-wall contributions of the fields to entropy and thermal energy. When the former is set to be equal to the black hole entropy by fixing the cutoff α as (3.20), the later becomes comparable with the mass of the background geometry. After that, it has been shown that at the Hawking temperature the wall contribution to the thermal energy is exactly canceled by the negative energy of the Boulware state, assuming implicitly that the ground state of the model is the Boulware state and that the

gravitational energy appearing in the Einstein equation is a sum of the renormalized energy of the Boulware state and the thermal energy of the fields.

In this section we shall show that these implicit assumptions do hold. In the following arguments it will also become clear how the local description used in the previous section is derived from the quantum field theory in curved spacetime, which is globally defined.

For simplicity, we consider a real scalar field described by the action

$$I = -\frac{1}{2}\int d^4x\sqrt{-g}\left[g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + m_\phi^2\phi^2\right]. \tag{4.1}$$

On the background given by (3.1), the action is reduced to

$$I = \int dtL, \tag{4.2}$$

with the Lagrangian L given by

$$L = -\frac{1}{2}\int d^3x r^2\sqrt{\Omega}\left[-\frac{1}{f}(\partial_t\phi)^2 + f(\partial_r\phi)^2 + \frac{1}{r^2}\Omega^{ij}\partial_i\phi\partial_j\phi + m_\phi\phi^2\right]. \tag{4.3}$$

Here x^i $(i = 1, 2)$ are coordinates on the 2-sphere. In order to make our system finite let us suppose that two mirror-like boundaries are placed at $r = r_1$ and $r = L$ $(r_1 < L)$, respectively, and investigate the scalar field in the region between the two boundaries. In the following arguments we quantize the scalar field with respect to the Killing time t. Hence, the ground state obtained below is the Boulware state. After the quantization, we investigate the statistical mechanics of the scalar field in the Boulware state. It will be shown that the resulting statistical mechanics is equivalent to the brick wall model.

Now let us proceed to the quantization procedure. First, the momentum conjugate to $\phi(r, x^i)$ is

$$\pi(r, x^i) = \frac{r^2\sqrt{\Omega}}{f}\partial_t\phi, \tag{4.4}$$

and the Hamiltonian is given by

$$H = \frac{1}{2}\int d^3x\left[\frac{f}{r^2\sqrt{\Omega}}\pi^2 + r^2\sqrt{\Omega}f\left(\partial_r\phi\right)^2 + \sqrt{\Omega}\Omega^{ij}\partial_i\phi\partial_j\phi + r^2\sqrt{\Omega}m_\phi^2\phi^2\right]. \tag{4.5}$$

Next, promote the field ϕ to an operator and expand it as

$$\phi(r,x^i) = \sum_{nlm} \frac{1}{\sqrt{2\omega_{nl}}} \left[a_{nlm}\varphi_{nl}(r)Y_{lm}(x^i)e^{-i\omega_{nl}t} + a^\dagger_{nlm}\varphi_{nl}(r)Y_{lm}(x^i)e^{i\omega_{nl}t}\right], \tag{4.6}$$

where $Y_{lm}(x^i)$ are real spherical harmonics defined by

$$\frac{1}{\sqrt{\Omega}}\partial_i\left(\sqrt{\Omega}\Omega^{ij}\partial_j Y_{lm}\right) + l(l+1)Y_{lm} = 0,$$
$$\int Y_{lm}(x^i)Y_{l'm'}(x^i)\sqrt{\Omega(x^i)}d^2x = \delta_{ll'}\delta_{mm'},$$

and $\{\varphi_{nl}(r)\}$ $(n = 1, 2, \cdots)$ is a set of real functions defined below, which is complete with respect to the space of L_2-functions on the interval $r_1 \le r \le L$ for each l. The positive constant ω_{nl} is defined as the corresponding eigenvalue.

$$\begin{aligned}\frac{1}{r^2}\partial_r\left(r^2 f\partial_r\varphi_{nl}\right) - \left[\frac{l(l+1)}{r^2} + m_\phi^2\right]\varphi_{nl} + \frac{\omega_{nl}^2}{f}\varphi_{nl} = 0,\\ \varphi_{nl}(r_1) = \varphi_{nl}(L) = 0,\\ \int_{r_1}^{L}\varphi_{nl}(r)\varphi_{n'l}(r)\frac{r^2}{f(r)}dr = \delta_{nn'}.\end{aligned} \tag{4.7}$$

The corresponding expansion of the operator $\pi(r,x^i)$ is then:

$$\pi(r,x^i) = -i\frac{r^2\sqrt{\Omega(x^i)}}{f(r)}\sum_{nlm}\sqrt{\frac{\omega_{nl}}{2}}\left[a_{nlm}\varphi_{nl}(r)Y_{lm}(x^i)e^{-i\omega_n t} - a^\dagger_{nlm}\varphi_{nl}(r)Y_{lm}(x^i)e^{i\omega_{nl}t}\right]. \tag{4.8}$$

Hence, the usual equal-time commutation relation

$$\left[\phi(r,x^i), \pi(r',x'^i)\right] = i\delta(r-r')\delta^2(x^i - x'^i) \tag{4.9}$$

becomes

$$\begin{aligned}\left[a_{nlm}, a^\dagger_{n'l'm'}\right] &= \delta_{nn'}\delta_{ll'}\delta_{mm'},\\ \left[a_{nlm}, a_{n'l'm'}\right] &= 0,\\ \left[a^\dagger_{nlm}, a^\dagger_{n'l'm'}\right] &= 0.\end{aligned} \tag{4.10}$$

The normal-ordered Hamiltonian is given by

$$: H := \sum_{nlm} w_{nl}a^\dagger_{nlm}a_{nlm}. \tag{4.11}$$

Thus, the Boulware state $|B\rangle$, which is defined by

$$a_{nlm}|B\rangle = 0 \tag{4.12}$$

for $^{\forall}(n, l, m)$, is an eigenstate of the normal-ordered Hamiltonian with the eigenvalue zero. The Hilbert space of all quantum states of the scalar field is constructed as a symmetric Fock space on the Boulware state, and the complete basis $\{|\{N_{nlm}\}\rangle\}$ $(N_{nlm} = 0, 1, 2, \cdots)$ is defined by

$$|\{N_{nlm}\}\rangle = \prod_{nlm} \frac{1}{\sqrt{N_{nlm}!}} \left(a^{\dagger}_{nlm}\right)^{N_{nlm}} |B\rangle, \tag{4.13}$$

and each member of the basis is an eigenstate of the normal-ordered Hamiltonian:

$$: H : |\{N_{nlm}\}\rangle = \left(\sum_{nlm} \omega_{nl} N_{nlm}\right) |\{N_{nlm}\}\rangle. \tag{4.14}$$

Now we shall investigate the statistical mechanics of the quantized scalar field. The free energy F is given by

$$e^{-\beta_\infty F} \equiv \mathbf{Tr}\left[e^{-\beta_\infty :H:}\right] = \prod_{nlm} \frac{1}{1 - e^{-\beta_\infty \omega_{nl}}}, \tag{4.15}$$

where $\beta_\infty = T_\infty^{-1}$ is inverse temperature. For explicit calculation of the free energy we adopt the WKB approximation. First, we rewrite the mode function $\varphi_{nl}(r)$ as

$$\varphi_{nl}(r) = \psi_{nl}(r) e^{-ikr}, \tag{4.16}$$

and suppose that the prefactor $\psi_{nl}(r)$ varies very slowly:

$$\left|\frac{\partial_r \psi_{nl}}{\psi_{nl}}\right| \ll |k|, \qquad \left|\frac{\partial_r^2 \psi_{nl}}{\psi_{nl}}\right| \ll |k|^2. \tag{4.17}$$

Thence, assuming that

$$\left|\frac{\partial_r (r^2 f)}{r^2 f}\right| \ll |k|, \tag{4.18}$$

the field equation (4.7) of the mode function is reduced to

$$k^2 = k^2(l, \omega_{nl}) \equiv \frac{1}{f}\left[\frac{\omega_{nl}^2}{f} - \frac{l(l+1)}{r^2} - m_\phi^2\right]. \tag{4.19}$$

Here we mention that the slowly varying condition (4.17) can be derived from the condition (4.18) and viceversa. The number of modes with frequency less than ω is given approximately by

$$\tilde{g}(\omega) = \int \nu(l,\omega)(2l+1)dl, \tag{4.20}$$

where $\nu(l,\omega)$ is the number of nodes in the mode with (l,ω):

$$\nu(l,\omega) = \frac{1}{\pi}\int_{r_1}^{L} \sqrt{k^2(l,\omega)}dr. \tag{4.21}$$

Here it is understood that the integration with respect to r and l is taken over those values which satisfy $r_1 \le r \le L$ and $k^2(l,\omega) \ge 0$. Thus, when

$$\left|\frac{\partial_r(r^2 f)}{r^2 f}\right| \ll \frac{1}{f\beta_\infty}$$

is satisfied, the free energy is given approximately by

$$F \simeq \frac{1}{\beta_\infty}\int_0^\infty \ln\left(1 - e^{-\beta_\infty \omega}\right)\frac{d\tilde{g}(\omega)}{d\omega}d\omega = \int_{r_1}^{L} \tilde{F}(r) 4\pi r^2 dr, \tag{4.22}$$

where the 'free energy density' $\tilde{F}(r)$ is defined by

$$\tilde{F}(r) \equiv \frac{1}{\beta(r)}\int_0^\infty \ln\left(1 - e^{-\beta(r)E}\right)\frac{4\pi p^2 dp}{(2\pi)^3}. \tag{4.23}$$

Here the 'local inverse temperature' $\beta(r)$ is defined by the Tolman's law

$$\beta(r) = f^{1/2}(r)\beta_\infty, \tag{4.24}$$

and E is defined by $E = \sqrt{p^2 + m_\phi^2}$. Hence the total energy U (equal to ΔM_{therm} given by (3.8)) and entropy S are calculated as

$$U \equiv \mathbf{Tr}\left[e^{\beta_\infty(F - :H:)} : H :\right] = \frac{\partial}{\partial \beta_\infty}(\beta_\infty F) = \int_{r_1}^{L} \rho(r) 4\pi r^2 dr, \tag{4.25}$$

$$S \equiv -\mathbf{Tr}\left[e^{\beta_\infty(F - :H:)} \ln e^{\beta_\infty(F - :H:)}\right] = \beta_\infty^2 \frac{\partial}{\partial \beta_\infty} F = \int_{r_1}^{L} s(r) 4\pi r^2 dr/\sqrt{f(r)}, \tag{4.26}$$

where the 'density' $\rho(r)$ and the 'entropy density' $s(r)$ are defined by

$$\rho(r) \equiv \frac{\partial}{\partial\beta(r)}(\beta(r)\tilde{F}(r)) = \int_0^\infty \frac{E}{e^{\beta(r)E}-1}\frac{4\pi p^2 dp}{(2\pi)^3},$$
$$s(r) \equiv \beta^2(r)\frac{\partial}{\partial\beta(r)}\tilde{F}(r) = \beta(r)\left(\rho(r)+P(r)\right), \tag{4.27}$$

where the 'pressure' $P(r)$ is defined by [1]

$$P(r) \equiv -\tilde{F}(r) = \frac{1}{3}\int_0^\infty \frac{p^2/E}{e^{\beta(r)E}-1}\frac{4\pi p^2 dp}{(2\pi)^3}. \tag{4.28}$$

These expressions are exactly same as expressions (3.5) for the local quantities in the statistical mechanics of gas of particles.

Thus, we have shown that the local description of the statistical mechanics used in Sec. III is equivalent to that of the quantized field in the curved background, which is defined globally, and whose ground state is the Boulware state.

The stress energy tensor of the minimally coupled scalar field is given by

$$T_{\mu\nu} = -\frac{2}{\sqrt{-g}}\frac{\delta I}{\delta g^{\mu\nu}} = \partial_\mu\phi\partial_\nu\phi - \frac{1}{2}g_{\mu\nu}\left(g^{\rho\sigma}\partial_\rho\phi\partial_\sigma\phi + m_\phi^2\phi^2\right). \tag{4.29}$$

In particular, the (tt)-component is

$$T^t_t = -\frac{1}{2}\left[\frac{1}{f}(\partial_t\phi)^2 + f(\partial_r\phi)^2 + \frac{1}{r^2}\Omega^{ij}\partial_i\phi\partial_j\phi + m_\phi^2\phi^2\right] \tag{4.30}$$

Hence, the contribution ΔM of the scalar field to the mass of the background geometry is equal to the Hamiltonian of the field:

$$\Delta M \equiv -\int_{r_1}^{L} T^t_t 4\pi r^2 dr = H, \tag{4.31}$$

where H is given by (4.5). When we consider the statistical mechanics of the hot quantized system, contributions of both vacuum polarization and thermal excitations must be taken into account. Thus, the contribution to the mass is given by

$$\langle\Delta M\rangle = \mathbf{Tr}\left[e^{\beta_\infty(F-:H:)}\Delta M^{(ren)}\right], \tag{4.32}$$

---

[1] To obtain the last expression of $P(r)$ we performed an integration by parts.

where $\Delta M^{(ren)}$ is an operator defined by the expression (4.31) with T_t^t replaced by the renormalized stress energy tensor $T^{(ren)t}{}_t$. From (4.31), it is easy to show that

$$\Delta M^{(ren)} =: H : +\Delta M_B, \tag{4.33}$$

where $: H :$ is the normal-ordered Hamiltonian given by (4.11) and ΔM_B is the zero-point energy of the Boulware state defined by

$$\Delta M_B = -\int_{r_1}^{L} \langle B|T^{(ren)t}{}_t|B\rangle 4\pi r^2 dr. \tag{4.34}$$

Hence, $\langle \Delta M\rangle$ can be decomposed into the contribution of the thermal excitations and the contribution from the zero-point energy:

$$\langle \Delta M\rangle = U + \Delta M_B, \tag{4.35}$$

where U is given by (4.25) and equal to ΔM_{therm} defined in (3.8).

Finally, we have shown that the gravitational mass appearing in the Einstein equation is the sum of the energy of the thermal excitation and the mass-energy of the Boulware state. Therefore, as shown in Sec. III, the wall contribution to the total gravitational mass is zero on shell ($T_\infty = T_H$) and the backreaction can be neglected. Here, we mention that the corresponding thermal state on shell is called a topped-up Boulware state [24], and can be considered as a generalization to spacetimes not necessarily containing a black hole of the Hartle-Hawking state [25].

V. SUMMARY AND DISCUSSION

Attempts to provide a microscopic explanation of the Bekenstein-Hawking entropy S_{BH} initially stemmed from two quite different directions. (See [26] for an up-to-date review with full references.)

Gibbons and Hawking [5] took the view that S_{BH} is of topological origin, depending crucially on the presence of a horizon. They showed that S_{BH} emerges as a boundary contribution to the geometrical part of the Euclidean action. (A non-extremal horizon

is represented by a regular point in the Euclidean sector, so the presence of a horizon corresponds to the *absence* of an inner boundary in this sector.)

'tHooft [13] sought the origin of S_{BH} in the thermal entropy of ambient quantum fields raised to the Hawking temperature. He derived an expression which is indeed proportional to the area, but with a diverging coefficient which has to be regulated by interposing a "brick wall" just above the gravitational radius and adjusting its altitude by hand to reproduce S_{BH} with the correct coefficient.

In addition, the brick wall model appears to have several problematical features—large thermal energy densities near the wall, producing a substantial mass correction from thermal excitations—which have raised questions about its self-consistency as a model in which gravitational back-reaction is neglected.

We have shown that such caveats are seen to be unfounded once the ground state of the model is identified correctly. Since there are no horizons above the brick wall, the ground state is the Boulware state, whose negative energy almost exactly neutralizes the positive energy of the thermal excitations. 'tHooft's model is thus a perfectly self-consistent description of a configuration which to outside observers appears as a black hole but does not actually contain horizons.

It is a fairly widely held opinion (e.g. [22,27]) that the entropy contributed by thermal excitations or entanglement is a one-loop correction to the zero-loop (or "classical") Gibbons-Hawking contribution. The viewpoint advocated in this paper appears (at least superficially) quite different. We view these two entropy sources—(a) brick wall, no horizon, strong thermal excitations near the wall, Boulware ground state; and (b) black hole, horizon, weak (Hartle-Hawking) stress -energy near the wall, Hartle-Hawking ground state—as ultimately equivalent but mutually exclusive (complementary in the sense of Bohr) descriptions of what is externally virtually the same physical situation. The near-vacuum experienced by free-falling observers near the horizon is eccentrically but defensibly explainable, in terms of the description (a), as a delicate cancellation between a large thermal energy and an equally large and negative ground-state energy—just as the Minkowski vacuum is explainable to a

uniformly accelerated observer as a thermal excitation above his negative-energy (Rindler) ground state. (This corresponds to setting $f(r) = r$ in the $(1+1)$-dimensional example treated in Sec. II.)

That the entropy of thermal excitations can single-handedly account for S_{BH} without cutoffs or other *ad hoc* adjustments can be shown by a thermodynamical argument [24]. One considers the reversible quasi-static contraction of a massive thin spherical shell toward its gravitational radius. The exterior ground state is the Boulware state, whose stress-energy diverges to large negative values in the limit. To neutralize the resulting backreaction, the exterior is filled with thermal radiation to produce a "topped-up" Boulware state (TUB) whose temperature equals the acceleration temperature at the shell's radius. To maintain thermal equilibrium (and hence applicability of the first law) the shell itself must be raised to the same temperature. The first law of thermodynamics then shows that the shell's entropy approaches S_{BH} (in the non-extremal case) for essentially arbitrary equations of state. Thus, the (shell + TUB) configuration passes smoothly to a black hole + Hartle-Hawking state in the limit.

It thus appears that one has two complementary descriptions, (a) and (b), of physics near an event horizon, corresponding to different Fock representations, i.e., different definitions of positive frequency and ground state. The Bogoliubov transformation that links these representations is known [28]. However, because of the infinite number of field modes, the two ground states are unitarily inequivalent [29]. This signals some kind of phase transition (formation of a condensate) in the passage between description (a), which explains S_{BH} as a thermal effect, and description (b), which explains it as geometry. We know that a condensation actually does occur at this point; it is more usually called gravitational collapse.

It will be interesting to explore the deeper implications of these connections.

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APPENDIX A: ON-SHELL BRICK WALL MODEL

When we performed the differentiation with respect to β_∞ to obtain the total energy and the entropy, the surface gravity κ_0 of the black hole and the inverse temperature β_∞ of gas on the black hole background were considered as independent quantities. Since in equilibrium these quantities are related by $\beta_\infty^{-1} = \kappa_0/2\pi$, we have imposed this relation, which we call the on-shell condition, after the differentiation. In fact, we have shown that the wall contribution to gravitational energy is zero and the backreaction can be neglected, if and only if the on-shell condition is satisfied.

On the other hand, in the so-called on-shell method [21,15], the on-shell condition is implemented before the differentiation. Now let us investigate what we might call an on-shell brick wall model. With the on-shell condition, the wall contribution to the free energy of the scalar field considered in Sec. IV is calculated as

$$F_{wall}^{(on-shell)} = -\frac{A}{4}\frac{\beta_\infty^{-1}}{360\pi}\frac{1}{\alpha^2}. \tag{A1}$$

If we define total energy and entropy in the on-shell method by

$$U_{wall}^{(on-shell)} \equiv \frac{\partial}{\partial \beta_\infty}\left(\beta_\infty F_{wall}^{(on-shell)}\right),$$
$$S_{wall}^{(on-shell)} \equiv \beta_\infty^2 \frac{\partial}{\partial \beta_\infty} F_{wall}^{(on-shell)}, \tag{A2}$$

then these quantities can be calculated as

$$U_{wall}^{(on-shell)} = 0,$$
$$S_{wall}^{(on-shell)} = \frac{A}{4}\frac{1}{360\pi}\frac{1}{\alpha^2} = \frac{1}{4}S_{wall}, \tag{A3}$$

where S_{wall} is the wall contribution (3.14) to entropy of the scalar field with $T_\infty = T_H$.

It is notable that the total energy $U_{wall}^{(on-shell)}$ in the on-shell method is zero irrespective of the value of the cutoff α. However, $S_{wall}^{(on-shell)}$ is always smaller than S_{wall}. It is because some physical degrees of freedom are frozen by imposing the on-shell condition before the differentiation. Thus, we might miss the physical degrees of freedom in the on-shell method.

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